Dear C&C Families,

Several years ago, we undertook to explore some key City and Country practices and policies in greater depth and to put them in writing for our families. We’re continuing this practice with our Spring 2010 issue of Currents.

Math work at City and Country School may not be as obvious to you as other areas of study because we so often integrate it into the social studies or Job for each Group. We are committed to designing the Math Program to foster a deep understanding of math concepts, mastery of problem-solving techniques and a real-world understanding of math and its role in our everyday lives.

For you to have a clear picture of math at C&C, this issue provides a snapshot of our approach, an in-depth description of math teaching and learning, and a section on refinements we have undertaken in the past few years to articulate and enhance our Math Program.

If you have any comments about math or this issue of Currents, please feel free to speak with me.

Sincerely,

Kate Turley
Principal
Math at C&C: Snapshot

As John Dewey pointed out long ago, learning is not either-or when it comes to conceptual and technical understanding. Our Math Program seeks to join the two in a way that makes sense for the learner, enabling him or her to use math as a tool for practical problem solving in school and in life.

Foundations
Our goal is to help children develop their intuitive mathematical thinking and then learn facts and techniques when the time is right. Real-world problem solving lies at the heart of true math inquiry; manipulative materials, traditional skills practice, and many teacher-made materials and games are used to reinforce concepts. We encourage children at all ages to explore different ways to find answers, to describe their mathematical thinking, and to compare and discuss their observations and approaches to solving problems. When all children are asked to talk about math and comment on their thinking in a problem-solving situation, math becomes an active process.

Context
In every area of study, children learn best in context. The City and Country School mathematics program is designed to provide that context, through social studies-related materials and work, such as blocks, and the Jobs Program.

Math-Rich Areas of C&C Curriculum
BLOCKS Lower School children often recreate their experiences and observations through unit blocks, which are all fractional units or multiples of one another. Blocks present many opportunities for learning mathematical concepts such as numbers (counting), symmetry and patterns (arrangement of blocks), mapping and measurement (planning and building structures), sorting and classifying (choosing block shapes and reshelving), spatial orientation (arrangements of structures), multiplication (grouping blocks when counting), fractions (observing the relationships among blocks), and geometry.

JOBS In the Middle and Upper School, the Jobs Program is one area that provides children with further opportunities to apply their math skills in real-life settings. Addition, subtraction, multiplication and division are solidified through the Post Office and Store work. Understanding of statistics, ratio and geometry concepts are developed through sign-making and with the printing presses. Data collection and statistical analysis are used in newspaper reporting. By the time the XIlls graduate, they have completed the equivalent of Algebra I, with an understanding of rational and irrational numbers and the ability to solve linear and exponential equations with variables.

Standards
How do we know what math topics to focus on at each age? The NCTM (National Council for Teachers of Mathematics) Focal Points are nationally accepted standards for the what and how of math, Kindergarten through 8th Grade. They "comprise related ideas, concepts, skills, and procedures that form the foundation for understanding and using mathematics". When first developed, these standards revolutionized the teaching of math by stressing the teaching of concepts in a practical, meaningful way, rather than focusing on facts and memorization as the primary way of learning. What their recommendations stress is what City and Country teachers have been practicing for decades. We keep our practice informed by their publications and updates, based on new research and teaching and learning techniques.

What is a real-world problem?
It is not a word problem (a routine exercise used to give context to a concept after it’s been taught), but a real-world problem, in which math concepts help children solve a problem they have already encountered. For example, when researching items to sell in the IXs Store, the children use their previous knowledge of ratio and multiplication to find the “best deal” from various vendors. It becomes clear to them that there must be a more efficient way to make their decisions: long division. This is the time that IXs teachers introduce the concepts of long division, while the children practice their new skills in a context that has real implications for their daily work.

*NCTM Focal Points:
http://www.nctm.org/standards/content.aspx?id=270
In every area of learning—reading, writing, social studies (including social relationships with one another), science and math, we think children learn best in context. For real learning to occur—and by that we mean sustained, internalized learning—children need to understand the purpose of it, to feel connected to it, to be invested in it. That is the beauty of our program. The core of what we do here is to provide opportunities for real experiences by which children can learn from the consequences of their actions.

**Lower School**

Each teacher has a clear idea of what mathematical concepts are appropriate and important for the children in his or her age group to work on. It’s the teacher’s job, then, to find many opportunities—within the context of the program whenever possible—for children to experience those concepts. For example, IIs and IIIs should be learning about comparisons such as “empty/full,” “more/less” and “short/long.” The ability to recognize and make comparisons is a basic mathematical concept that helps form the foundation for later learning. You will often hear IIs and IIIs teachers questioning children about such things. At the water table a teacher might say, “Your bowl was full. Now it’s empty. Which other bowls are full?” In the Lower School, there are many possibilities for this type of contextual learning—so much is embedded in the everyday life of the program and the basic materials; every action in a classroom carries an opportunity for learning. Some examples: Young children are taught to pour a half of a cup of juice for snack (i.e. fractions), and as snack and lunchtime helpers—part of the rotation on classroom job charts—children count out the proper amounts of cups, juice and food. When Vs are tallying the number of milks for the Group, they have to factor in that one milk serves two children.

The unit blocks found in Lower School classrooms are all fractional units or multiples of one another, and through repeated use children learn important mathematical concepts, including numbers, symmetry and patterns, mapping and measurement, sorting and classifying, spatial orientation, fractions, multiplication and geometry. Walking around the Lower School classrooms, you will see many examples of math in action during block building. A two-year-old builds a train of brickies, each topped with a cubie in a pattern of yellow, blue, yellow, blue. A four-year-old has fifteen towers of butteries and roundies in a row, each carefully balanced, with three blocks per stack. A seven-year-old runs out of longies and must figure out how many she needs to borrow from another Group to complete her bridge. They’re out of longies, too? How many middlies can I borrow? The significance of blocks is that the math involved is not just numbers but relationships, too. Caroline Pratt knew what she was doing.

The blocks themselves present many opportunities for learning important mathematical concepts, including numbers, symmetry and patterns, mapping and measurement, sorting and classifying, spatial orientation, fractions, multiplication and geometry.
It is through repeated exploration and experimentation with the materials that children come to intuit and hone these concepts. Real math learning doesn’t happen overnight. Because children at C&C have ample time to play and ruminate with the materials and repeated experiences working with them, the learning becomes internalized—a real part of the child. At the youngest ages, the children are not yet able to express the concepts in language or manipulate them on paper. Later, however, the concepts are taught in a more formal way. For example, when “fractions” are taught at VII, the children are given the language and symbols for the concept they already understand intuitively.

Similarly, children are taught the standard algorithms for efficiency (as when you add the ones first, then the tens), but only after they’ve had lots of concrete experiences with numbers to ensure that solid understanding is in place. A child who doesn’t fully grasp the idea of tens and ones—an abstract concept—and that the digit “3” in the number “35” actually means “30” and not “3,” will not make heads or tails of adding the ones first, then the tens. If they do not understand this, imagine their confusion when you tell them to “borrow from the tens.”

Middle and Upper School
In the Middle and Upper School, the Jobs Program provides children with further opportunities to apply their math skills in real-life settings. Calculating transactions and making change in the VIII’s Post Office reinforces two- and three-digit addition and subtraction as well as multiplication facts, whereas a wider variety of supplies in the IX’s Store requires the children to have solid multiplication and division skills at their fingertips. The XIs deepen their study of proportionality and ratios through their use of the printing presses, and the XIIIs often conduct statistical research and report the findings in the Newspaper.

A solid foundation of arithmetic, multiplication and division, as well as the basics of geometry are in place by the IXs. The Xs and XIs children are exploring geometric shapes using protractors and three-dimensional scale models, and two- and three-step problem-solving schemes involving fractions, decimals, and percentages are developed and refined. In the XIIIs, the children execute simple proofs using Euclidean geometry. By the time they graduate, they have completed the equivalent of Algebra I, with an understanding of rational and irrational numbers and the ability to solve linear and exponential equations with variables.

Teaching and Learning Techniques
Each year builds on previous years. We believe strongly in the importance of laying a good foundation of mathematical thinking and problem solving when children are very young—even two! Our goal is to help children develop their intuitive mathematical thinking and then learn facts and techniques when the time is right. To this end, we use many manipulative materials, some more traditional skills practice exercises and many teacher-made materials and games. Games are a perfect way to reinforce concepts. A variety of approaches is used in order to cement the children’s understanding.

We encourage children at all ages to explore many different ways to find answers. We also encourage children to describe their mathematical thinking and to compare and discuss their observations and approaches to solving problems. It’s in the process of exploring that children learn. They need to understand the complexity and flexibility of numbers and that there isn’t only one way to find the answer. Given that people by nature use many different methods of math strategizing to find answers, if we were to teach children math in just one way, we would exclude so many ways of thinking! In addition, all of this talking about and describing math thinking is a way for children to learn from each other. Someone else’s idea might make perfect sense to you, but you might never have thought of it yourself.

Do you know how to measure a building with your shadow and a ruler?
Using their knowledge of similar triangles, XIIIs measure their own shadow and their own height. They then measure the length of a C&C building shadow and using the triangle created by their body, and their knowledge of ratio, they can then calculate the height of the building. The XIIIs then test their work by dropping a string from the top of the building to the Yard.
It’s very important to us to try to break the stereotype of girls as “mathphobic” or girls perceiving themselves as not being as good at math as boys. Gender research shows a solution to math gender bias is to include more problem solving and discussion in the math curriculum from the earliest ages. When children are all asked to talk about math and comment on their thinking in a problem-solving situation, it helps them to establish their mathematical voice. Teachers at C&C make it their business to draw out all voices. A quiet voice becomes a louder one with repeated expectations to participate. We give children ample opportunity to attach a voice to their math thinking. Math is active; children are not sitting passively with workbooks. And, because there are also rules in the classrooms about not calling out answers during discussions, children are given time to think before answers are blurted out. In this way, the child who needs more time to think has a chance to participate—and does not feel defeated by the members of the Group who always have that quick answer. After all, some of the best, most complex thinking can take time to develop.

Assessment
How do teachers assess children’s math abilities? The fact that children are consistently encouraged to explain their thinking helps teachers know exactly what the children understand in far more detail than a simple test. Teachers observe, listen and question on a daily basis. They adjust instruction and plan activities based on what they see and hear. Teachers are always asking questions such as, Why do you think that? How can you prove that? Does your answer make sense? As children get older, writing is incorporated into the Math Program. At VII, the children keep math journals. Teachers and children alike benefit from this process. Writing asks us to organize and clarify our thinking, and in the process, we deepen our understanding.

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Math Refinements: Recent Years

Over the past few years C&C has focused on refining our Math Program for greater continuity across the ages, to best reflect recent math learning research, and to mindfully prepare our graduates for advanced math study in high school.

Curriculum Continuity
The Lower School teachers have been improving continuity of the Math Program across the Vs, VIs and VIIIs. They have conducted summer curriculum grant work and collaborative school-year work that includes explicit outlining of the major goals of those years. Knowing that we build on children’s knowledge at each age, we want to be sure they have a solid and deep understand every step of the way. Some schools use a spiral curriculum model where certain concepts are touched on lightly at more than one age, whereas we prefer to go deeper with fewer concepts at each age, thus allowing for a nuanced understanding of the mathematical concept and techniques before moving on to another.

Along similar lines, the Middle and Upper School has taken a hard look at the math curriculum progression in light of what a graduate will need to place out of Algebra 1 in his or her freshman year of high school. Using the standards supplied by competitive and typical high schools attended by our graduates, Gino Crocetti, Ann Roberts and Juan Carlos Infante worked through the Middle and Upper School Math Program to make sure the requirements for placement out of Algebra 1 are thoughtfully embedded in our curriculum, in an appropriate sequence, without compromising our approach.

Social Studies and Math
As discussed previously, City and Country is dedicated to exploring math in real-world contexts and allowing the need for the math to arise first, and then teaching and learning the most efficient math concepts available to the age group as a way of solving a real problem. Our Lower School teachers have also used summer curriculum grants to further articulate the real-world connections between social studies and mathematics. VIs Teacher Erin Teesdale outlined a number of social studies contexts that are well-suited for math study. For example, a walk around the block for the VIs neighborhood study yields opportunities to observe shape and pattern, and to practice measurement, data collection, and mapping. The VIs food study might also lead to collecting, graphing and analyzing data about a variety of subtopics, such as the kinds of foods found in their homes and health and dental health topics, such as teeth lost. Within this study they also have the opportunity to explore geometry through drawing and recreating foods and their shapes. Restaurants and food also provide an excellent context for working with money math (addition and subtraction), and refining a variety of measurement and estimation skills while cooking.

How do blocks and social studies provide context for children when learning math concepts?

VIIIs Teacher Molly Lippman explains that direct conversation and reflection about a block pattern (blocks in relation to one another), on a tower of the Brooklyn Bridge helps a child take the leap from knowing that it looks “cool” to having an explicit and deeper understanding of the mathematical concepts: two squaries plus one brickie plus one middlie equal one longie!

There are many methods for reflecting on the math and geometry of blocks during pick-up time. Some of Molly’s examples: Estimate how many blocks you used in your building. How did you make your estimate? Stack—reevaluate your estimate. Put away a few stacks keeping track of how many blocks you’ve put away. Reevaluate again.

A teacher holds up a longie and asks the children to put away all of the blocks that are ¼ of a longie.

She also suggests that when studying New York City children can extend a learned fact by exploring a question, such as: “The Empire State Building went up at a rate of 4 ½ stories per week—how many stories would be up in two weeks?”
Taking advantage of salient contexts for math learning becomes more challenging in the oldest ages and our teachers have focused on developing areas in which real-world needs are explored through math. XIIIIs, who study public health issues in connection with their American history social studies, have worked on statistical projects using C&C attendance and health data. Xs have also used statistical mapping of the heights of the children in the School to determine the best placement of signs on the walls for greatest visibility.

**Research-Based Enhancements**

At C&C, our program accommodates a variety of learning styles and abilities. Our teachers have been working specifically on refining this element of math learning. This “differentiated” learning approach is well-suited to our context-rich math program, which allows a child to work with an idea in a meaningful way, and from a variety of angles, while also allowing children to go deeper and further, as their interest and ability allows.

In light of new research, we have been revising some teaching methods and Robert Berkman, C&C Math Consultant, has written to parents in detail about some changes to our program. Some highlights include his explanation that rapid-fire testing of math facts does not necessarily reflect a student’s understanding of math, instead it shows us how accurately he or she recalls math facts under stress. In fact, research shows that this kind of memorization is a linguistic skill and testing it tells us more about linguistic memorization ability than it does about mastery of a math computation. And, among many other things, he has introduced a more flexible and transparent algorithm for learning long division, which allows us greater insight into a child’s thinking.

Finally, over the past few years, teachers have been working with children on math concepts and problem-solving techniques during “math lunch” where children can elect to eat lunch together in a classroom and then work on a challenging yet playful math problem in small groups. This activity is designed to give a new perspective on a math concept in an interesting, open-ended and hands-on environment.

**Conclusion**

Math at City and Country School, like all of our work here, provides a deep learning experience. Being able to master a computational technique of a mathematical concept is too shallow. It is a deep, far-reaching understanding of mathematics that is our goal. Just as decoding language is the base for reading, a computational technique is the technical base for mathematics. With decoding you can read “Stop!,” but you can’t interpret Homer. Likewise, through real-world problem-solving and nuanced exploration of math concepts, children at C&C learn to fully comprehend math.

*Edited by Jennifer Marck Moran*